MTH 201: Multivariable Calculus and Differential Equations

Problem Set 1: 3d geometry

1. Find the vector, parametric, and symmetric equations for the following lines.

   (a) The line through points $(6, 1-3)$ and $(2, 4, 5)$.

   (b) The line through $(2, 1, 0)$ and perpendicular to both $x = y$ and $y = z$.

   (c) The line of intersection of $x + y + z = 1$ and $x + z = 0$.

   (d) The line through $(1, -1, 1)$ and parallel to $x + 2 = \frac{y}{2} = z - 3$.

   (e) The line through $(5, 1, 0)$ and perpendicular to $2x - y + z = 1$.

   (f) The line through $(-4, -6, 1)$ and $(2, 5, 3)$, perpendicular to the line through $(-3, 2, 0)$ and $(5, 1, 4)$.

2. Find the vector and scalar equations of the following planes.

   (a) The plane through $(-2, 8, 10)$ and perpendicular to $x = 1 + t, y = 2t, z = 4 - 3t$.

   (b) The plane through $(4, -2, 3)$ and parallel to $3x - 7z = 12$.

   (c) The plane through $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$.

   (d) The plane through $(-1, 2, 1)$ and contains the line of intersection of $x+y-z = 2$ and $2x - y + 3z = 1$.

   (e) The plane that passes through the line of intersection of $x - z = 1$ and $y + 2z = 3$ and is perpendicular to $x + y - 2z = 1$.

   (f) The plane that contains $x = 3+2t, y = t, z = 8-t$ and is parallel to $2x+4y+8z = 17$.

3. Show that the distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by
$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

4. Determine whether the following planes are parallel, perpendicular, or neither. If parallel, find the distance between them, and if neither, find the angle between them.

   (a) $x + 2y + 2z = 1$, $2x - y + 2z = 1$.

   (b) $2x - 3y + 4z = 5$, $x + 6y + 4z = 3$.

   (c) $x = 4y - 2z$, $8y = 1 + 2x + 4z$.

5. Dertermine whether the lines L_1 and L_2 line are parallel, skew, or intersecting. If they intersect, find the point of intersection.

   (a) L_1: $x = 1 + 2t, y = 3t, z = 2 - t$
   L_2: $x = -1 + s, y = 4 + s, z = 1 + 3s$

   (b) L_1: $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$
   L_2: $\frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$

   (c) L_1: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
   L_2: $\frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$

   (d) L_1: $x = -6t, y = 1 + 9t, z = -3t$
   L_2: $x = 1 + 2s, y = 4 - 3s, z = s$

6. Find the traces (cross sections) of the given quadric surfaces in the planes $x = k$, $y = k$, $z = k$. Then identify the surfaces and sketch them.

   (a) $y = z^2 - x^2$

   (b) $y^2 - z^2 - x^2 = 1$

   (c) $4x - y^2 + 4z^2 = 0$

   (d) $x = 2y^2 + 3z^2$

   (e) $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$

   (f) $4x^2 + 9y^2 = 36z^2 = 36$.

7. Sketch the following surfaces in $\mathbb{R}^3$.

   (a) $yz = 4$.

   (b) $z = e^y$.

   (c) $z = \cos x$.

   (d) $z = y^2 + xy$.